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UNIDOMINATING FUNCTION OF ROOTED PRODUCT $P_m \circ C_n$

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ABSTRACT: Dominating functions in domination theory of graphs have interesting applications. The theory of domination in graphs was introduced by Ore [7] and Berge [1]. The concepts of dominating functions are introduced by Hedetniemi [6]. Rooted product graphs is a new concept introduced by Godsil [5] has become an inviting area of research at present. Anantha Lakshmi [15] has introduced new concepts of unidominating function of a graph and studied these functions for some standard graphs. In this paper the authors have presented unidominating function and unidomination number for rooted product graph of a path and cycle graph.

KEYWORDS: Unidominating function, Unidomination number, rooted product graph

Classification number: 05C69, 05C76

INTRODUCTION: For general notation and basic concepts of graph theory we refer to Berge [1]. All graphs mentioned in this paper are simple, non-trivial, connected, and finite graphs unless mentioned otherwise. In 1978, Godsil and Mckay [5] introduced a new product of two graphs G and H, called rooted product denoted by G $\odot$ H. The rooted product of a graph G and a rooted graph H is obtained by taking |V(G)| copies of H, and for every vertex v_i of G, identifying v_i with the root node of the i-th copy of H. The Rooted product of a path P_m with a rooted cycle graph C_n is a graph obtained by taking one copy of a m-vertex graph P_m and m-copies of C_n, this graph is denoted as $P_m \odot C_n$.

V Ananthalakshmi [15] has introduced the concept of unidominating function.

Let G(V,E) be a graph. A function $f: V \rightarrow \{0,1\}$ is said to be a unidominating functions if

$\sum_{u\in N[v]} f(u) \geq 1$ and $f(v) = 1$

$\sum_{u\in N[v]} f(u) = 1$ and $f(v) = 0$

The unidomination number of a graph G (V,E) is

$\gamma_u(G) =$ min{f(V)/ f is a unidominating function} where f (V) = $\sum_{u\in V} f(u)$.

V .Ananthalakshmi [15] found that the unidomination number of path P_m is $\gamma_u(P_m) = \left\lceil \frac{m}{3} \right\rceil$.

Rashmi S B[13] has proved that the unidomination number of cycle C_n is $\gamma_u(C_n) = \left\lceil \frac{n}{3} \right\rceil \; for\; n \equiv 0,1(mod 3)$

$$= \left\lceil \frac{n}{3} \right\rceil + 1 \quad for\; n \equiv 2(mod 3)$$

UNIDOMINATING FUNCTION OF $P_m\, o\, C_n$

In this section we find the unidominating function of minimum weight on $P_m\ o\ C_n$ and hence find its unidomination number.

Theorem: Unidominating function of rooted product of $P_m \circ C_n$ is

$$\gamma_u(P_m \circ C_n) = \begin{cases} X + k + r_1 a + \left\lceil \frac{r_1}{2} \right\rceil & for\; m \equiv 0,1,2(mod3), n \equiv 0(mod3) \\ X + r_1 a + \left\lceil \frac{r_1}{2} \right\rceil & for\; m \equiv 0,1,2(mod3), n \equiv 1(mod3) \\ X + m + r_1 a + \left\lceil \frac{r_1}{2} \right\rceil & form \equiv 0,1,2(mod3), n \equiv 2(mod3) \end{cases}$$

Where X= k(3a+1) , m=3k+r_1 and n= 3a+r_2

Proof: Consider rooted product graph $P_m \circ C_n$, Let the vertex set of path graph P_m be $V(P_m) = \{v_1, v_2, v_3, \dots \dots \dots . v_m\}$ and vertex set of the cycle graph C_n be $V(C_n) = \{u_1, u_2, u_3, \dots \dots \dots . u_n\}$. Let the root vertex from cycle C_n be u_1(any other vertex chosen as root will not change the proof).

So the vertex set of rooted product graph becomes

$$V(P_m \circ C_n) = \{(u_i, v_j); i = 1,2, \dots . . m \; \& \; j = 1,2, \dots \dots n\}$$

With root vertex set in $P_m \circ C_n$ as

$$roots(P_m \circ C_n) = \{(u_1, v_1), (u_1, v_2), (u_1, v_3), \dots\dots\dots (u_1, v_m)\}$$

For this root vertex set is a path graph, we use the Unidominating function defined in the paper of V. Anantha Lakshmi and B. Maheshwari [15] given below.

$$f(u_i) = 1 \quad \text{when } m \equiv 0\ (mod 3)$$

$$= \left\lceil \frac{m}{3} \right\rceil \quad \text{when } m \equiv 1\ (mod 3)$$

$$= 2 \quad \text{when } m \equiv 2\ (mod 3)$$

We extend this definition of function for the rooted product graph, by defining the function values on the vertices of m-copies of cycle graphs, $P_{n-1} = C_n -($ root vertex). Similar to the function definition on path on (n-1) vertices, gives that $\gamma_u(P_m \circ C_n) \geq \gamma_u(P_m) + m\,\gamma_u(P_{n-1})$.

Let us write $X = \gamma_u(P_m) + m\,\gamma_u(P_{n-1})$ so we have $\gamma_u(P_m \circ C_n) \geq X$.

Now, the adjacencies due to connectivity of root vertex in the path as well as in the cycle graph, we need to check for uni-dominating condition satisfied for all vertices (u_i, v_j) for which $f(u_i, v_j) = 0$.

Case (I) : For $m \equiv 0(mod 3)$

For m=3k, the 2k vertices $(u_1, v_1), (u_1, v_3), (u_1, v_4), (u_1, v_6) \dots\dots$ are assigned the function value zero. The remaining k vertices $(u_1, v_2), (u_1, v_5), (u_1, v_8), \dots ....$ are assigned the function value one. For the copy of C_n attached to these vertices, (n-1) cycle vertices (u_2, v_j),$(u_3, v_j), \dots\dots\dots\dots.(u_n, v_j)$ attached to the vertex (u_1, v_j) for j=1,2,……m as follows.

From [15] we define the function for the path vertices in $P_m \circ C_n$ as ,

$$f(u_i, v_j) = \begin{cases} 1\ for\ i \equiv 2(mod 3) \\ 0\ for\ i \equiv 0,1(mod 3) \end{cases}$$

Subcase (IA): For $n \equiv 0(mod 3)$ Let n=3a $\Longrightarrow a = \frac{n}{3}$

Now when $f((u_i, v_j) = 0$, we define the function for the cycle graph vertices as,

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$$f((u_i,v_j) = \begin{cases} 1 \ \ for\ i \equiv 0(mod3) and\ i = n-1, n-2 \\ 0 \qquad for\ i \equiv 1,2(mod3) and\ i = n \end{cases}$$

Next when $f(u_i,v_j) = 1$ then to satisfy the unidominating condition, we define

$$f(u_i,v_j) = \begin{cases} 1\ for\ i \equiv 1(mod3) \\ 0\ for\ i \equiv 0,2(mod3) \end{cases}$$

We check the unidominating condition for all vertices at which function value is assigned zero.

(I) On Path:

(i) $f(u_1,v_j) = f(u_1,v_{j-1}) + f(u_1,v_j) + fu_1,v_{j+1}) + f(u_2,v_j) + f(u_n,v_j)$

= 1+0+0+0+0=1

(ii) $f(u_1,v_1) = f(u_2,v_1) + f(u_1,v_1) + +f(u_1,v_2) + f(u_n,v_1)$=0+0+0+1=1

(iii) $f(u_1,v_m) = f(u_1,v_{m-1}) + f(u_1,v_m) + fu_2,v_m) + f(u_n,v_m)$ =1+0+0+0=1

(II) On Cycle :

(iv) $f(u_i,v_j) = f(u_{i-1},v_j) + f(u_i,v_j) + fu_{i+1},v_j) = 0+0+1 = 1$

(v) $f(u_n,v_m) = f(u_1,v_m) + f(u_n,v_m) + fu_{n-1},v_m)$ =0+0+1=1

As at all five cases when $f(u_i,v_j) = 0$, $\sum_{u \in N} f = 1$ we get that for

$m \equiv 0(mod3)$ & $n \equiv 0(mod3)$ the function satisfies unidominating condition with weight of the function f(V) equal to

f(V) = $\sum_{i=1}^{n} \sum_{j=1}^{m} f(u_i,v_j)$

= 2k(0+0+1+0+0+1+………+1+1+0)+k(1+0+0+1+0+0+…….+1+0+0)

= 2k (a+1) + k a

=3ka+2k

f(V) = k(3a+1) +k =X +k

Example 1:

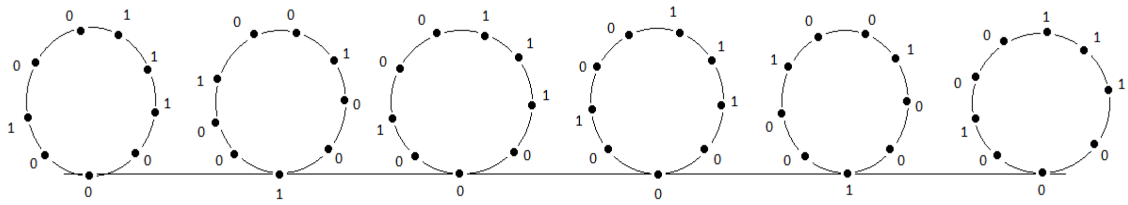

Fig 1: $\gamma_u(P_6 \circ C_9) = 22$

Subcase (IB) : For $n \equiv 1(mod3)$ Let n=3a+1 $\Rightarrow a = \frac{n-1}{3}$

Now when $f(u_i, v_j) = 0$, we define the function for the cycle graph vertices as,

$$f(u_i, v_j) = \begin{cases} 1 & for\ i \equiv 0(mod3) \\ 0 & for\ i \equiv 1,2(mod3) \end{cases}$$

Next when $f(u_i, v_j) = 1$ then to satisfy the unidominating condition, we define

$$f(u_i, v_j) = \begin{cases} 1 & for\ i \equiv 1(mod3) and\ \ i = n \\ 0 & for\ i \equiv 0,2(mod3) \end{cases}$$

We check the unidominating condition for all vertices at which function value is assigned zero.

(I) On Path:

$(i)\ f(u_1, v_j) = f(u_1, v_{j-1}) + f(u_1, v_j) + fu_1, v_{j+1}) + f(u_2, v_j) + f(u_n, v_j)$

= 1+0+0+0+0=1

$(ii) f(u_1, v_1) = f(u_2, v_1) + f(u_1, v_1) + f(u_n, v_1) + f(u_1, v_2)$=0+0+0+1=1

$(iii) f(u_1, v_m) = f(u_1, v_{m-1}) + f(u_1, v_m) + fu_2, v_m) + f(u_n, v_m)$ =1+0+0+0=1

(II) On Cycle :

(iv) $f(u_i, v_j) = f(u_{i-1}, v_j) + f(u_i, v_j) + fu_{i+1}, v_j) = 0 + 0 + 1 = 1$

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(v) $(f(u_n, v_m) = f(u_{n-1}, v_m) + f(u_n, v_m) + fu_1, v_m)$ =1+0+0=1

As at all five cases when $f(u_i, v_j) = 0$, $\sum_{u\in N} f = 1$ we get that for $m \equiv 0(mod3)$ & $n \equiv 0(mod3)$ the function. The function f satisfies unidominating condition with weight of the function f(V) equal to

f(V) = $\sum_{i=1}^{n}\sum_{j=1}^{m} f(u_i, v_j)$

= 2k (0+0+1+0+0+1+…+0+0+1+0)+k(1+0+0+1+0+0+…….+1+0+0+1)

=2k a $+ k(a + 1)$

=3ak +k

f(v) = k(3a+1) =X

Example 2:

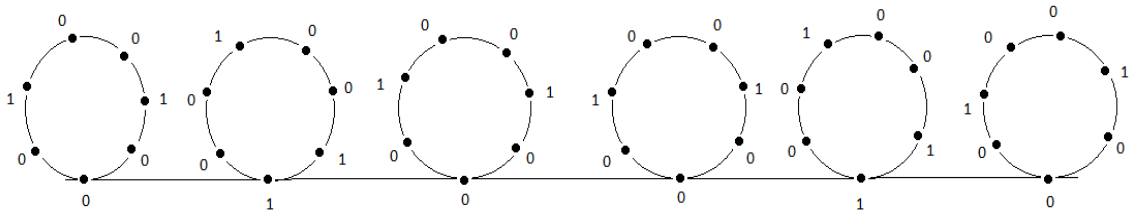

Fig 2: $\boldsymbol{\gamma_u(P_6 \circ C_7) = 14}$

Subcase (IC): For $n \equiv 2(mod3)$ Let n=3a+2 $\Longrightarrow a = \frac{n-2}{3}$

Now when $f(u_i, v_j) = 0$, we define the function for the cycle graph vertices as,

$$f(u_i, v_j) = \begin{cases} 1 & for\ i \equiv 0(mod3)\ and\ i = n-1 \\ 0 & for\ i \equiv 1,2(mod3) \end{cases}$$

Next when $f(u_i, v_j) = 1$ then to satisfy the unidominating condition , we define

$$f(u_i, v_j) = \begin{cases} 1 & for\ i \equiv 1(mod3) and\ \ i = n \\ 0 & for\ i \equiv 0,2(mod3) \end{cases}$$

We check the unidominating condition for all vertices at which function value is assigned zero.

(I) On Path:

(i) $f(u_1, v_j) = f(u_1, v_{j-1}) + f(u_1, v_j) + fu_1, v_{j+1}) + f(u_2, v_j) + f(u_n, v_j)$

= 1+0+0+0+0=1

$(ii)f(u_1, v_1) = f(u_2, v_1) + f(u_1, v_1) + f(u_n, v_1) + f(u_1, v_2)$=0+0+0+1=1

$(iii)f(u_1, v_m) = f(u_1, v_{m-1}) + f(u_1, v_m) + fu_2, v_m) + f(u_n, v_m)$ =1+0+0+0=1

(II) On Cycle :

(i) $f(u_i, v_j) = f(u_{i-1}, v_j) + f(u_i, v_j) + fu_{i+1}, v_j) = 0 + 0 + 1 = 1$

(v) $f(u_n, v_m) = f(u_{n-1}, v_m) + f(u_n, v_m) + fu_1, v_m)$ =1+0+0=1

As at all five cases when $f(u_i, v_j) = 0$, $\sum_{u \in N} f = 1$ we get that for $m \equiv 0(mod3)$ & $n \equiv 0(mod3)$ the function .The function f satisfies unidominating condition with weight of the function f(V) equal to

f(V) = $\sum_{i=1}^{n} \sum_{j=1}^{m} f(u_i, v_j)$

= 2k (0+0+1+0+0+1+…+0+0+1+0)+k(1+0+0+1+0+0+…….+1+1)

=2k $[a + 1] + k(a + 2)$

=3ka+k+3k

(IC) f(V)= k(3a+1)+3k =X+m

Example 3:

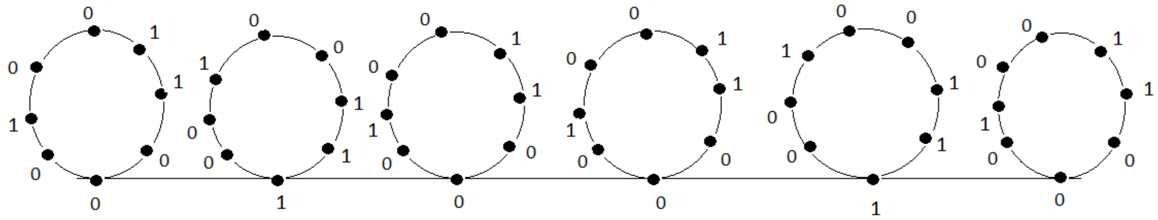

Fig 3 : $\gamma_u(P_6 \circ C_8) = 20$

Case (II): For $m \equiv 1(mod3)$ Let m=3k+1 $\Longrightarrow k = \frac{m-1}{3}$

From [15] we define the function for the path vertices in $P_m \circ C_n$ as ,

$$f(u_i, v_j) = \begin{cases} 1 \;\; for\ i \equiv 2(mod3)\ and\ i = n-1 \\ 0 \;\; for\ i \equiv 0{,}1(mod3) \end{cases}$$

Subcase (IIA): For $n \equiv 0(mod3)$ Let n=3a $\Longrightarrow a = \frac{n}{3}$

Now when $f(u_i, v_j) = 0$, we define the function for the cycle graph vertices as,

$$f(u_i, v_j) = \begin{cases} 1 \quad for\ i \equiv 0(mod3) \\ 0 \quad for\ i \equiv 1{,}2(mod3) \end{cases}$$

Next when $f(u_i, v_j) = 1$ then to satisfy the unidominating condition, we define

$$f(u_i, v_j) = \begin{cases} 1 \;\; for\ i \equiv 1(mod3) \\ 0 \;\; for\ i \equiv 0{,}2(mod3) \end{cases}$$

We check the uni domination condition for all vertices at which function value is assigned zero.

As the function is identical to the definition given in subcase (IA) except at values j= m, m-1, m-2, m-3 for i=1. Therefore, we check only the unidominating condition for these values i=1, j=m. m-3 only when the function value is zero.

$$f(u_1, v_{m-3}) = f(u_1, v_{m-4}) + f(u_1, v_{m-3}) + fu_1, v_{m-2}) + f(u_2, v_{m-3}) + f(u_n, v_{m-3})$$

=0+0+1+0+0=1

$f(u_1, v_m) = f(u_1, v_{m-1}) + f(u_1, v_m) + fu_2, v_m) + f(u_n, v_m)$ =1+0+0+0=1

Hence the unidominating condition satisfied with weight of the function

f(V) = $\sum_{i=1}^{n} \sum_{j=1}^{m} f(u_i, v_j)$

= 2k (0+0+1+0+0+1+…+1+1+0) +(k+1)(1+0+0+1+0+0+…….+1+0+0)

=2k (a+1) $+(k+1)\,a$ = k(3a+1)+k+a

(IIA) f(V)=X+k+a

Example 4:

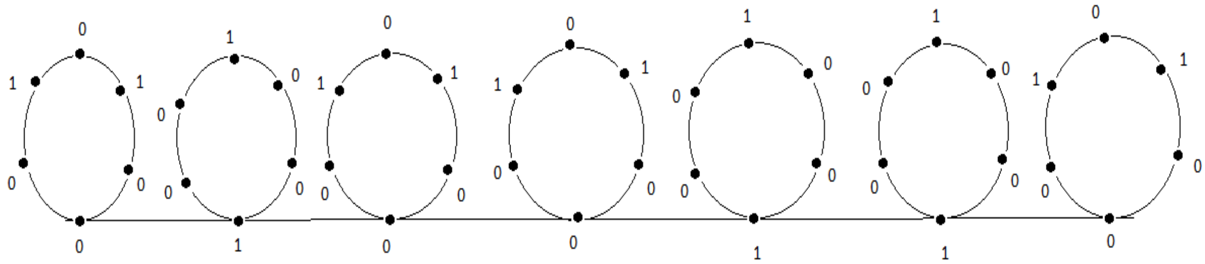

Fig 4: $\gamma_u(P_7 \circ C_6) = 14$

Subcase (IIB) : For $n \equiv 1(mod3)$ Let n=3a+1 $\Longrightarrow a = \frac{n-1}{3}$

Now when $f(u_i, v_j) = 0$, we define the function for the cycle graph vertices as,

$$f(u_i, v_j) = \begin{cases} 1 & for\ i \equiv 0(mod3) \\ 0 & for\ i \equiv 1,2(mod3) \end{cases}$$

Next when $f(u_i, v_j) = 1$,we define the function for the cycle graph vertices as,

$$f(u_i, v_j) = \begin{cases} 1 & for\ i \equiv 1(mod3) \\ 0 & for\ i \equiv 0,2(mod3) \end{cases}$$

As the function is identical to the definition given in subcase (IIA) except at values n,n-2 for i=1. Therefore we check only the uni domination condition for these values i when the function value is zero.

$$f(u_n, v_j) = f(u_1, v_j) + f(u_n, v_j) + fu_{n-1}, v_j) = 0 + 0 + 1 = 1$$

$$f(u_{n-2}, v_j) = f(u_{n-1}, v_j) + f(u_{n-2}, v_j) + fu_{n-3}, v_j) = 1 + 0 + 0 = 1$$

Hence the unidominating condition satisfied with weight of the function f(V) = $\sum_{i=1}^{n}\sum_{j=1}^{m} f(u_i, v_j)$

= 2k (0+0+1+0+0+1+…++0+0+1+0) +(k+1)(1+0+0+1+0+0+…+1+0+0+1)

=2k a $+(k+1)(a+1)$ = k (3a+1)+a+1

(II B) f(V) =X+a+1

Example 5:

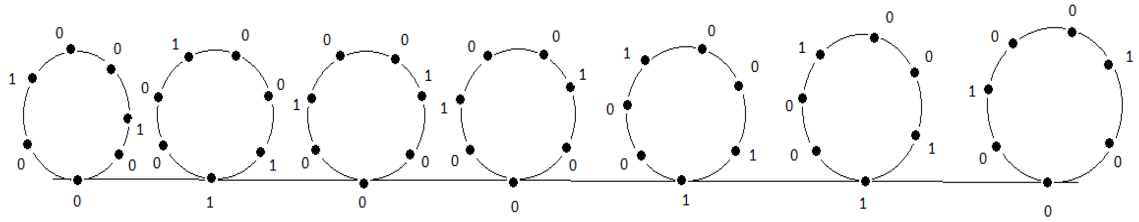

Fig 5 : $\boldsymbol{\gamma_u(P_7 \circ C_7) = 17}$

Subcase (IIC) : For $n \equiv 2(mod3)$ Let n=3a+2 $\Longrightarrow a = \frac{n-2}{3}$

From [15] we define the function for the path vertices in $P_m \circ C_n$ as ,

$$f(u_i, v_j) = \begin{cases} 1 \ \ for\ i \equiv 2(mod3)\ and\ i = n-1 \\ 0 \ \ for\ i \equiv 0,1(mod3) \end{cases}$$

Now when $f(u_i, v_j) = 0$, we define the function for the cycle graph vertices as,

$$f(u_i, v_j) = \begin{cases} 1 \ \ \ for\ i \equiv 0(mod3)\ \ and\ i = n-1 \\ 0 \ \ for\ i \equiv 1,2(mod3) \end{cases}$$

Next when $f(u_i, v_j) = 1$, we define the function for the cycle graph vertices as,

$$f(u_i, v_j) = \begin{cases} 1 \ \ for\ i \equiv 1(mod3)\ and\ i = n \\ 0\ for\ i \equiv 0,2(mod3) \end{cases}$$

As the function is identical to the definition given in subcase (IIB) except at values j$\not\equiv$ $2(mod3)$ n, n-3. Therefore, we check only the unidominating condition for j$\not\equiv$ $2(mod3)\ and\ i =$ n, n-3, when the function value is zero.

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$$f(u_n, v_j) = f(u_1, v_j) + f(u_n, v_j) + fu_{n-1}, v_j) = 0 + 0 + 1 = 1$$

$$f(u_{n-3}, v_j) = f(u_{n-2}, v_j) + f(u_{n-3}, v_j) + fu_{n-4}, v_j) = 1 + 0 + 0 = 1$$

Hence the uni domination condition satisfied with weight of the function

f(V) = $\sum_{i=1}^{n} \sum_{j=1}^{m} f(u_i, v_j)$

= 2k (0+0+1+0+0+1+…+1+0) +(k+1)(1+0+0+1+0+0+…+1+1)

=2k [a+1] $+ (k+1)(a+2)$

=k(3a+1)+(3k+1)+ +a+1

(IIC) f(V)= X+m+a+1

Example 6:

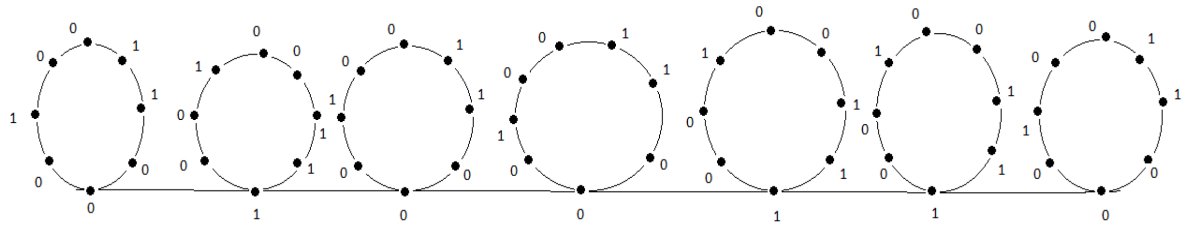

Fig 6 : $\gamma_u(P_7 \circ C_8) = 24$

Case (III): For $m \equiv 2(mod3)$ Let m=3k+2 $\Longrightarrow k = \frac{m-2}{3}$

From [15] we define the function for the path vertices in $P_m \circ C_n$ as ,

$$f(u_i, v_j) = \begin{cases} 1\ for\ i \equiv 2(mod3) \\ 0\ \ for\ i \equiv 0,1(mod3) \end{cases}$$

Subcase (IIIA) : For $n \equiv 0(mod3)$ Let n=3a $\Longrightarrow a = \frac{n}{3}$

Now when $f(u_i, v_j) = 0$, we define the function for the cycle graph vertices as,

$$f(u_i, v_j) = \begin{cases} 1\ \ \ for\ i \equiv 0(mod3) and\ i = n-1, n-2 \\ 0\ \ for\ i \equiv 1,2(mod3) \end{cases}$$

Next when $f(u_i, v_j) = 1$ then to satisfy the unidominating condition, we define

$$f(u_i, v_j) = \begin{cases} 1 & for\ i \equiv 1(mod3) \\ 0 & for\ i \equiv 0,2(mod3) \end{cases}$$

As the function is identical to the definition given in subcase (IA) except at values j= m, m-1, m-2, m-3 for i=1. Therefore, we check only the uni domination condition for these values i=1, j=m-1, m-2 only when the function value is zero.

$$f(u_1, v_{m-1}) = f(u_1, v_{m-2}) + f(u_1, v_{m-1}) + fu_1, v_m) + f(u_2, v_{m-1}) + f(u_n, v_{m-1})$$

=0+0+1+0+0=1

$$f(u_1, v_{m-2}) = f(u_1, v_{m-3}) + f(u_1, v_{m-2}) + fu_1, v_{m-1}) + f(u_2, v_{m-2}) + f(u_n, v_{m-2})$$

=1+0+0+0+0=1

Hence the unidominating condition satisfied with weight of the function

f(V) = $\sum_{i=1}^{n} \sum_{j=1}^{m} f(u_i, v_j)$

= (2k+1) (0+0+1+0+0+1+…+1+1+0) +(k+1)(1+0+0+1+0+0+…….+1+0+0)

=(2k+1) [a+1] $+ (k+1)\,a$

=3ka+2k+2a+1

= k(3a+1)+k+2a+1

(IIIA) f(V) = X+k+2a+1

Example 7:

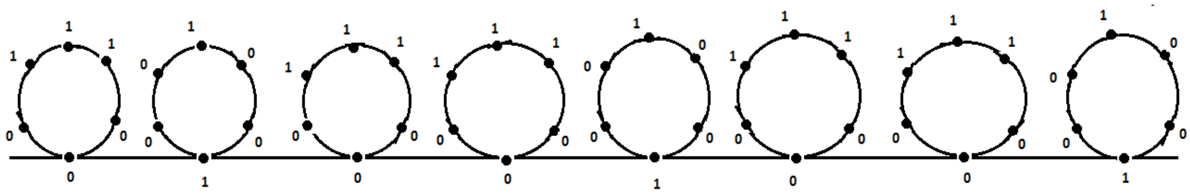

Fig 7 : $\gamma_u(P_8 \circ C_6) = 21$

Subcase (IIIB) : For $n \equiv 1(mod3)$ Let n=3a+1 $\Longrightarrow a = \frac{n-1}{3}$

Now when $f(u_i, v_j) = 0$, we define the function for the cycle graph vertices as,

$$f(u_i, v_j) = \begin{cases} 1 \; for \; i \equiv 0(mod3) \\ 0 \; for \; i \equiv 1,2(mod3) \end{cases}$$

Next when $f(u_i, v_j) = 1$ then to satisfy the unidominating condition, we define

$$f(u_i, v_j) = \begin{cases} 1 \; for \; i \equiv 1(mod3) \\ 0 \; for \; i \equiv 0,2(mod3) \end{cases}$$

As the function is identical to the definition given in subcase (IIIA) except at values j$\not\equiv$ $2(mod3)$ i =n, n-2. Therefore we check only the unidominating condition for these values for j$\not\equiv$ $2(mod3)$ &i= n, n-2only when the function value is zero.

$$f(u_n, v_j) = f(u_1, v_j) + f(u_n, v_j) + fu_{n-1}, v_j) = 0 + 0 + 1 = 1$$

$$f(u_{n-2}, v_j) = f(u_{n-1}, v_j) + f(u_{n-2}, v_j) + fu_{n-3}, v_j) = 1 + 0 + 0 = 1$$

Hence the unidominating condition satisfied with weight of the function

f(V) = $\sum_{i=1}^{n} \sum_{j=1}^{m} f(u_i, v_j)$

=(2k+1)(0+0+1+0+0+1+…+0+0+1+0) +(k+1)(1+0+0+1+0+0+…….+1+0+0+1)

=(2k+1) a+$(k + 1)(a + 1)$

= 3ka+k+2a+1

=k(3a+1)+2a+1

(IIIB) f(V) = X+2a+1

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Example 8:

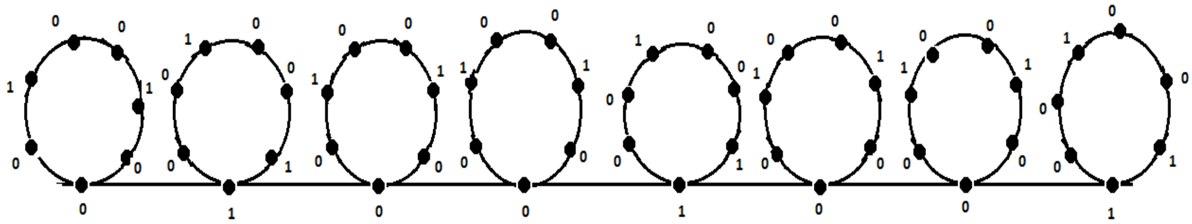

Fig 8 : $\boldsymbol{\gamma_u(P_8 \circ C_7) = 19}$

Subcase(IIIC) : For $n \equiv 2(mod3)$ Let n=3a+2 $\Longrightarrow a = \frac{n-2}{3}$

Now when $f((u_i, v_j) = 0,$ we define the function for the cycle graph vertices as,

$$f(u_i, v_j) = \begin{cases} 1\ for\ i \equiv 0(mod3)\ and\ i = n-1 \\ 0\ for\ i \equiv 1,2(mod3) \end{cases}$$

Next when $f((u_i, v_j) = 1$ then to satisfy the uni dominating condition, we define

$$f(u_i, v_j) = \begin{cases} 1\ \ for\ i \equiv 1(mod3)\ and\ i = n \\ 0\ \ for\ i \equiv 0,2(mod3) \end{cases}$$

As the function is identical to the definition given in subcase (IIIA) except at values j≢ $2(mod3)$ i =n,n-3. Therefore we check only the uni domination condition for these values for j≢ $2(mod3)$ &i= n,n-3only when the function value is zero.

$$f(u_n, v_j) = f(u_1, v_j) + f(u_n, v_j) + fu_{n-1}, v_j) = 0 + 0 + 1 = 1$$

$$f(u_{n-3}, v_j) = f(u_{n-2}, v_j) + f(u_{n-3}, v_j) + fu_{n-4}, v_j) = 1 + 0 + 0 = 1$$

Hence the uni domination condition satisfied with weight of the function

f(V) = $\sum_{i=1}^{n} \sum_{j=1}^{m} f(u_i, v_j)$

= (2k+1) (0+0+1+0+0+1+…+1+0) +(k+1)(1+0+0+1+0+0+…….+1+1)

= (2k+1) $[a + 1] + (k + 1)(a + 2)$

= 3ka +k+3k+2+2a+1 = k(3a+1)+(3k+2)+2a+1

(IIIC) f(V) = X+m+2a+1

Example 9:

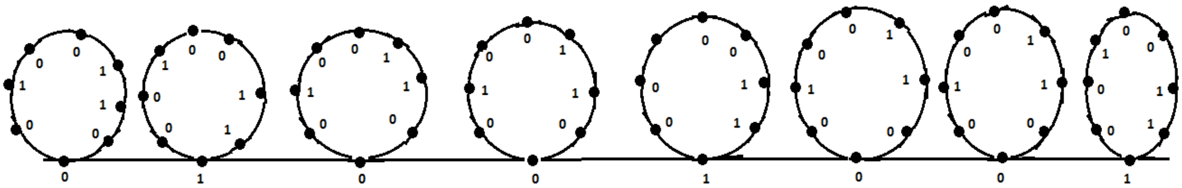

Fig 9 : $\gamma_u(P_8 \circ C_8) = 27$

Hence combining all the results from nine subcases together we can write unidomination number of rooted product of $P_m \circ C_n$ with X= k(3a+1) as given in below table

$\gamma_u(P_m \circ C_n)$	m=3k	m=3k+1	m=3k+2
n=3a	X+k	X+k+a	X+k+2a+1
n=3a+1	X	X+a+1	X+2a+1
n=3a+2	X+m	X+m+a+1	X+m+2a+1

Therefore we can write the unidomination number of rooted product as,

$$\gamma_u(P_m \circ C_n) = \begin{cases} X + k + r_1 a + \left\lfloor \frac{r_1}{2} \right\rfloor & for\ m \equiv 0,1,2(mod3), n \equiv 0(mod3) \\ X + r_1 a + \left\lceil \frac{r_1}{2} \right\rceil & for\ m \equiv 0,1,2(mod3), n \equiv 1(mod3) \\ X + m + r_1 a + \left\lceil \frac{r_1}{2} \right\rceil & form \equiv 0,1,2(mod3), n \equiv 2(mod3) \end{cases}$$

Where X= k(3a+1) , m=3k+r_1 and n= 3a+r_2

CONCLUSION: We have found unidomination number for rooted product of path and cycle graph with any one cycle vertex as root and we present the relation between unidomination number of product graph and its factors satisfying vizing conjecture

$$\gamma_u(P_m \circ C_n) \geq \gamma_u(P_m)\, \gamma_u(C_n)$$

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